Improved Mixed-Boundary Component-Mode Representation for Structural Dynamic Analysis
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(Short Version of Briefing)

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ASD's RFMB Derivation

- Derivation impetus: What if you “add” extra degrees of freedom to a Hurty/Craig-Bampton representation via residual flexibility <u>without</u> changing the normal mode boundary conditions?
- Derivation Achievements
  - Developed a generalized <u>mixed</u>-boundary dynamic math model reduction method that exactly reduces to the time-tested methods of:
    - Hurty/Craig-Bampton (all fixed-boundary, i.e., all B-set)
    - Rubin/MacNeal for the (all free-boundary, i.e., all C-set)
  - Demonstrated that Hurty/Craig-Bampton and Rubin/MacNeal are actually just <u>special</u> <u>cases</u> of a more general formulation
    - Hurty/Craig-Bampton and Rubin/MacNeal previously thought of as being completely distinct methods
  - Demonstrated excellent accuracy for all mixed-boundary combinations (B-set + C-set cases)
  - Demonstrated rapid convergence relative to the classic use of the “over-constrained” Hurty/Craig-Bampton formulation

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RFMB Coordinate Transformation

- Residual Flexibility Mixed-Boundary (RFMB) coordinate transformation
  - Utilizing standard NASTRAN set notation
  - Please see paper for full derivation details

$$\begin{Bmatrix} x_b \\ x_c \\ x_o \end{Bmatrix} = \begin{bmatrix} I_{bb} & 0 & 0 \\ 0 & I_{cc} & 0 \\ \psi_{ob}^{C} - {}^{R}g_{oc}\,{}^{R}g_{cc}^{-1}\,\psi_{cb}^{C} & {}^{R}g_{oc}\,{}^{R}g_{cc}^{-1} & \phi_{ok}^{N} - {}^{R}g_{oc}\,{}^{R}g_{cc}^{-1}\,\phi_{ck}^{N} \end{bmatrix} \begin{Bmatrix} x_b \\ x_c \\ q_k \end{Bmatrix}$$

Majed, A., Henkel, E. E., and Wilson, C., "Improved Method of Mixed-Boundary Component Representation for Structural Dynamic Analysis," AIAA Journal of Spacecraft and Rockets, Vol. 42, No. 5, September-October 2005.

RFMB Facts:

- Developed by Applied Structural Dynamics, Inc.
- Published in the peer-reviewed AIAA Journal of Spacecraft and Rockets (2005)
- Adopted by commercial finite element software including MSC/NASTRAN as default (2002)
  - PARAM, MHRED, NO to override default
- Solves the over-constrained Hurty/Craig-Bampton convergence issues
- Exactly reduces to Hurty/Craig-Bampton for all fixed-boundary (all B-set)
- Exactly reduces to Rubin-MacNeal for all free-boundary (all C-set)
- Highly accurate for all mixed-boundary cases (B-set + C-set)
- Proven to have no "rank deficiency" issues
- Simplifies component damping matrix development
- All parameters may be directly derived from test, resulting in a more rigorous definition of test/analysis correlation
- Utilized on mission critical Shuttle/payloads Verification Loads Cycles

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RFMB Coordinate Transformation (Cont'd)

- RFMB coordinate transformation exactly reduces to Hurty/Craig-Bampton for the all B-set case and Rubin/MacNeal for the all C-set case

$$\begin{Bmatrix} x_b \\ x_o \end{Bmatrix} = \begin{bmatrix} I_{bb} & 0 \\ \psi_{ob}^{C} & \phi_{ok}^{N} \end{bmatrix} \begin{Bmatrix} x_b \\ q_k \end{Bmatrix}$$

Hurty/Craig-Bampton

$$\begin{Bmatrix} x_b \\ x_c \\ x_o \end{Bmatrix} = \begin{bmatrix} I_{bb} & 0 & 0 \\ 0 & I_{cc} & 0 \\ \psi_{ob}^{C} - {}^{R}g_{oc} \, {}^{R}g_{cc}^{-1} \psi_{cb}^{C} & {}^{R}g_{oc} \, {}^{R}g_{cc}^{-1} & \phi_{ok}^{N} - {}^{R}g_{oc} \, {}^{R}g_{cc}^{-1} \phi_{ck}^{N} \end{bmatrix} \begin{Bmatrix} x_b \\ x_c \\ q_k \end{Bmatrix}$$

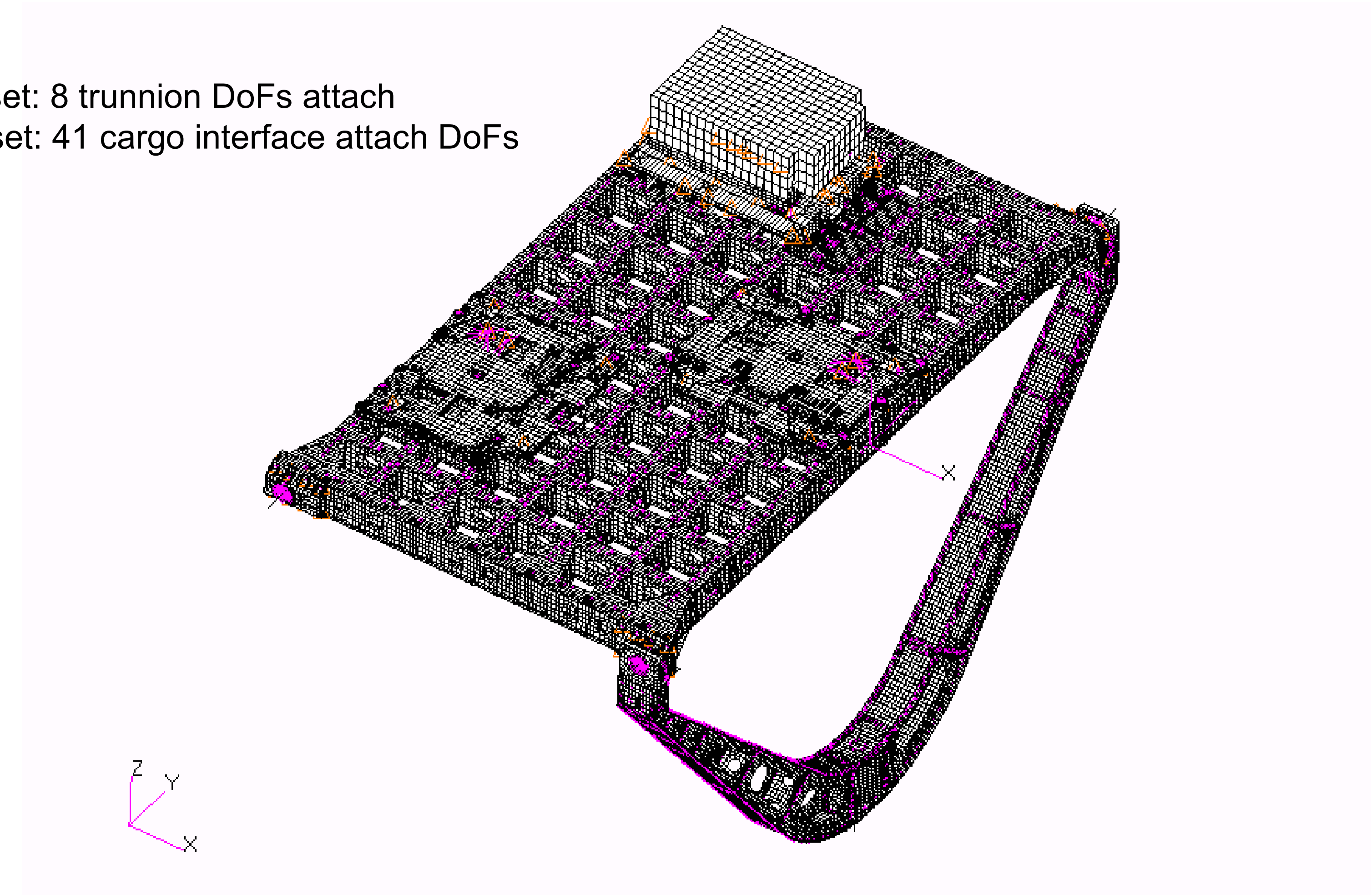
Residual Flex Mixed-Boundary

$$\begin{Bmatrix} x_c \\ x_o \end{Bmatrix} = \begin{bmatrix} I_{cc} & 0 \\ {}^{R}g_{oc} \, {}^{R}g_{cc}^{-1} & \phi_{ok}^{N} - {}^{R}g_{oc} \, {}^{R}g_{cc}^{-1} \phi_{ck}^{N} \end{bmatrix} \begin{Bmatrix} x_c \\ q_k \end{Bmatrix}$$

Rubin

Example – 650,000 dofs Integrated Cargo Carrier (ICC) Finite Element Model

B-set: 8 trunnion DoFs attach
C-set: 41 cargo interface attach DoFs



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Numerical Results: RFMB vs. Hurty/Craig-Bampton: Percent Frequency Error

Mode #	Exact Frequency (Hz)	RFMB	Guyan	H/CB +8 Modes (100 Hz)	H/CB +15 Modes (150 Hz)	H/CB +24 Modes (200 Hz)	H/CB +42 Modes (250 Hz)	H/CB +74 Modes (300 Hz)	H/CB +242 Modes (400 Hz)
1	18.06	0	0.79	0.53	0.33	0.02	0.01	0.01	0
2	21.29	0	2.94	0.15	0.07	0.04	0.03	0.02	0
3	30.07	0	2.53	1.39	0.42	0.09	0.04	0.03	0.01
4	32.87	0	5.16	1.23	0.46	0.08	0.06	0.05	0
5	47.77	0	32.52	0.37	0.11	0.04	0.02	0.01	0.01
6	50.4	0	34.7	0.42	0.14	0.04	0.03	0.02	0
7	53.7	0	71.87	1.16	0.33	0.14	0.09	0.05	0.02
8	59.67	0	69.6	1.37	0.36	0.22	0.11	0.06	0.02
9	67.25	0	74.04	3.01	1	0.23	0.13	0.06	0.04
10	82.43	0	48.09	4.7	1.42	0.97	0.55	0.34	0.12
11	84.01	0	53.48	7.78	0.71	0.24	0.17	0.11	0.05
12	86.65	0	52.11	6.08	2.05	1.31	0.78	0.53	0.17
13	90.93	0	63.6	9.44	3.02	0.74	0.33	0.21	0.12
14	92.91	0		13.26	1.77	0.53	0.36	0.19	0.08
15	101.94	0		16.91	3.13	2.86	2.75	1.56	0.23
16	105.69	0		18.69	1.64	1.21	0.99	0.17	0.03
17	108.07	0		19.83	4.65	3.36	2.82	0.76	0.21
18	113.95	0		19.27	7.57	3.82	3.35	1.89	0.29
19	119.36	0			7.72	4.87	2.28	1.41	0.44
20	130.08	0			1.26	0.68	0.42	0.21	0.09
21	131.51	0			7.96	0.86	0.51	0.33	0.15
22	139	0				2.94	1.64	0.9	0.41
23	141.07	0				2.99	1.26	0.78	0.39
24	147.62	0						1.58	0.76

Note: It took 242 modes and 4 hours and 56 minutes of computation time for Hurty/Craig-Bampton to converge to RFMB's accuracy level with just 24 modes. RFMB's execution time: 59 minutes. Both problems executed on same machine running MSC/NASTRAN

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Concluding Remarks

- RFMB is a generalization of Hurty/Craig-Bampton and Rubin/MacNeal
  - Bridges the gap between two methods previously thought to be completely unrelated
  - Exactly equal to Hurty/Craig-Bampton for the all-fixed-boundary case
  - Exactly equal to Rubin/MacNeal for the all-free-boundary case
  - Accurate and fast for mixed-boundary case
- RFMB utilized on critical programs such as Space Shuttle to develop dynamic math models for the Verification Loads Analyses (VLAs)
- RFMB affords tremendous flexibility and accuracy in component dynamic math model (DMM) development
  - e.g., greater modal convergence per number of reduced DMM degrees of freedom versus the classic use of over-constrained Hurty/Craig-Bampton formulation
- Default reduction method in MSC/NASTRAN
- Contact ASD if you have any questions:
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