An Improved Method of Structural-Borne Random Vibration Mass Attenuation

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Acknowledgements

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A special thanks to Dr. Terry Scharton for his review of the methodology and insightful inputs.
Objective

Provide a detailed description and demonstration of the Generalized Mass Attenuation (GMA) approach to random vibration mass attenuation
Motivation

In 2009, a vibroacoustics working group, formed as a sub-team of the NASA Engineering and Safety Center (NESC) Loads and Dynamics Technical Discipline Team, identified the need for improved random vibration mass attenuation prediction methods as a key area to reducing risk to new crew and launch vehicles due to vibroacoustic environment over- or under-specification.
Mass Attenuation

Problem Statement

**Given:** The structural-borne random vibration environment for the unloaded flight mounting structure $\alpha$

**Find:** The structural-borne random vibration environment due to the addition of component(s) $\beta$

Examples:
- Component $\alpha$: a launch vehicle's skin panel; a satellite's bus
- Component $\beta$: an avionics box mounted to the panel; a black-box mounted to the bus
Mass Attenuation
Approaches

• Base Shake?
  – Mile's Equation

• Computational
  – FEM/BEM

• Classical
  – Barrett
  – Norton-Thevenin
  – GMA
Base Shake

Base shake is **not** a method of mass attenuation. By definition, base shaking a component or a coupled system at its interfaces does **not** attenuate or amplify the same interface accelerations.

The RMS acceleration of a SDoF subjected to a white noise base shake (Mile’s equation) is often used to enforce random vibration acceleration criteria on components:

\[
\sigma_\ddot{x}^\beta = \sqrt{\frac{\pi}{2}} Q f_0 S_{\ddot{x}\ddot{x}}^\beta (f_0)
\]

\[
S_{\ddot{x}\ddot{x}}^\beta (f) = S_{\ddot{x}\ddot{x}}^\alpha (f)
\]

Notes:
- Derived from base shake of a SDoF with white noise acceleration input
- Provides an estimate for the RMS value of output acceleration
- Assumes **no** attenuations/amplifications due to component coupling
- “Infinite” Energy available to the analysis
- Typically very conservative
- Quick and simple
Computational

The computational methods rely on modeling (1) propagating acoustic field and acoustic/structure or (2) modeling the acoustic or TBL wall pressure. The coupled model is then solved and desired response items including component interface accelerations and forces are recovered.

Notes:
- Examples include FEM/BEM; FEM/FEM; FEM/SEA
- Linear acoustics (linearized wave equation)
- Idealized acoustic/structure coupling
- Idealized random field approximations for wall-pressure (Corcos, acoustic)
- Not a practical way to assess manifest variations
- Test verification requires coupled system
- Can be expensive and time intensive
Classical

The starting point for the classical mass attenuation methods is the component $\alpha$ interface acceleration environment. In this way, the complexities and nonlinearities associated with the pressure fields are “built-in”. Modeling of the fields or the wall pressures are avoided completely. However, these methods must properly account for the “finite energy” nature of the environment.

Notes:
- Examples: Barrett, Modified-Barrett, Norton-Thevenin, and GMA
- “Finite” energy approaches
- Simplified testing (component a only) to derive accelerations
- Highly practical way to assess manifest configurations
- Relatively simple calculations
Modified Barrett's Method

Single input/output equation for predicting the component loaded interface acceleration. Assumes constant “attenuation” across all frequencies. No interface amplifications are possible in Barrett's equation.

\[ S_{\ddot{x}}^{\alpha+\beta}(\omega) = A^2 S_{\ddot{x}}^{\alpha}(\omega) \]

\[ A = \frac{m^\alpha}{m^\alpha + m^\beta} \]

Note: the original Barrett's method has no exponent on the attenuation.
Norton-Thevenin (N-T)

Single input/output equation for predicting the component loaded interface acceleration. Improves on Barrett's by providing a frequency-dependent attenuation. N-T does allow for possible interface acceleration amplifications.

\[
S_{\ddot{x}\ddot{x}}(\omega) = |A(\omega)|^2 S_{\ddot{x}\ddot{x}}(\omega)
\]

\[
A(\omega) = \frac{m^\alpha(\omega)}{m^\alpha(\omega) + m^\beta(\omega)} = \frac{1/H^\alpha(\omega)}{1/H^\alpha(\omega) + 1/H^\beta(\omega)}
\]

Apparent Mass  Compliance FRF
GMA Acceleration Equation

The GMA interface acceleration equation presents a multi-drive point, multi-axis equation allowing for input environment correlations for improved prediction of the interface accelerations.

Fundamental principle enabling the derivation is that the structural-borne environment contains “finite energy” for driving the components.

GMA does not require simplifying assumptions relative to the environment or coupling.

The mathematical tools utilized for the derivation are modal synthesis and random vibration theories.

GMA affords a rational and robust approach in analyzing complex structural systems.
GMA

Acceleration Calculation Equation

Multi-drive point, multi-axis input/output equation for predicting the interface accelerations. Allows for input environment correlations for improved prediction of the interface acceleration attenuations/amplifications.

\[
S_{\dot{x}_k \dot{x}_l}^{\alpha + \beta} (\omega) = \sum_{i=1}^{b} \sum_{j=1}^{b} A_{ki}^{*} (\omega) A_{lj} (\omega) S_{\dot{x}_i \dot{x}_j}^{\alpha} (\omega)
\]

Interface DoF indices

“Drive point” & “transfer” Attenuations

Where the GMA attenuations terms \( A_{ij}(\omega) \) are computed as separate functions of flight mounting structure (\( \alpha \)) and component (\( \beta \)) interface compliances given by the equation:

\[
\begin{bmatrix} A_{bb}(\omega) \end{bmatrix} = \left( \left[ H_{bb}^{(\alpha)}(\omega) \right]^{-1} + \left[ H_{bb}^{(\beta)}(\omega) \right]^{-1} \right)^{-1} \left[ H_{bb}^{(\alpha)}(\omega) \right]^{-1}
\]

Interface compliance matrices
N-T and Barrett

Turn out to be Special Cases of GMA

\[ S^{\alpha+\beta}_{\ddot{x}_k \ddot{x}_l} (\omega) = \sum_{i=1}^{b} \sum_{j=1}^{b} A^*_k (\omega) A_j (\omega) S^{\alpha}_{\ddot{x}_i \ddot{x}_j} (\omega) \]

\[ S^{\alpha+\beta}_{\ddot{x} \ddot{x}} (\omega) = |A(\omega)|^2 S^{\alpha}_{\ddot{x} \ddot{x}} (\omega) \]

\[ S^{\alpha+\beta}_{\ddot{x} \ddot{x}} (\omega) = A^2 S^{\alpha}_{\ddot{x} \ddot{x}} (\omega) \]
GMA Demonstration Problem
Problem Statement

- Given the structural-borne random vibration environment for the empty Fuselage, predict the acceleration environment at the **Fuselage/Carrier** interfaces for the coupled system
  - Also, predict the **Carrier/Cargo** interface acceleration environment
Fuselage FEM

- Fuselage section FEM constructed:
  - 9896 lbs; 3965 natural frequencies between 20-2000 Hz analysis range; $\zeta = 2\%$
  - 7,986 DoFs, 1,273 elements
  - 7 DoFs for Carrier attach is shown

FEM utilized to derive Impedances for following Interface DoFs

Fuselage FEM
(Component $\alpha$)
Carrier FEM

- Carrier FEM constructed:
  - 2532 Lb (total); 281 natural frequencies: 20-2000 Hz analysis range; $\zeta = 2\%$
  - ~42,000 DoFs, 5192 elements
  - 7 DoFs for Fuselage attach is shown

Note: Carrier loaded on the STBD side with a 505 lb Cargo attached to the Carrier at 4 points, 3 DoFs per point

FEM utilized to derive Impedances for following Interface DoFs

Carrier FEM (Component $\beta$)
Fuselage Unloaded Environment

- Z axis environment is chosen since the indeterminacy in the Fuselage/Carrier interface in the Z-direction will display the subject methods ability to easily handle indeterminate interfaces without any simplifying assumptions

Orbiter Cargo Bay Random Vibration Longeron Environment: Table 4.1.6.2.3-1 Core ICD

<table>
<thead>
<tr>
<th>Z Axis</th>
<th>Range</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 – 45 Hz</td>
<td>0.009 G^2/Hz</td>
</tr>
<tr>
<td></td>
<td>45 – 70 Hz</td>
<td>+12 dB/Octave</td>
</tr>
<tr>
<td></td>
<td>70 – 600 Hz</td>
<td>0.050 G^2/Hz</td>
</tr>
<tr>
<td></td>
<td>600-2000 Hz</td>
<td>-6 dB/Octave</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>6.95 GRMS</td>
</tr>
</tbody>
</table>
Problem Solution

The interface attenuations for the GMA equation were computed from the interface impedances derived directly from the FEMs. A damping of 2% of critical was assumed.

The interface attenuations and the Fuselage unloaded environment provided the inputs to the GMA interface acceleration equation.

The GMA equation was then utilized for two different input correlation state assumptions: fully correlated and fully uncorrelated cases. The actual correlation state is somewhere between these two cases.
Fuselage/Carrier Interface Accelerations

Correlated Input Case

Note: Starboard side PSDs more attenuated due to cargo mass manifested on the starboard side.
Carrier/Cargo Interface Accelerations

Correlated Input Case

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![Graph showing carrier/cargo interface accelerations with labeled frequencies and accelerations.](image.png)

- **Orbiter Cargo Bay Zo**
  - Random Environment: 6.95 GRMS
- **FWD -Y Interface Accel. Zo**: 1.62 GRMS
- **FWD +Y Interface Accel. Zo**: 1.34 GRMS
- **AFT +Y Interface Accel. Zo**: 1.62 GRMS
- **AFT -Y Interface Accel. Zo**: 0.91 GRMS
Fuselage/Carrier Interface Accelerations

Uncorrelated Input Case

Note: Two closely spaced (27 Hz) un-symmetric roll and yaw modes with significant effective mass are strongly excited by the uncorrelated inputs case. This was not the case for correlated inputs, where the same un-symmetric modes remain essentially unexcited. Potential excitation of un-symmetric modes is an important feature of uncorrelated inputs.
Carrier/Cargo Interface Accelerations

Uncorrelated Input Case

- Orbiter Cargo Bay Zo Random Environment: 6.95 GRMS
- FWD -Y Interface Accel. Zo: 1.99 GRMS
- FWD +Y Interface Accel. Zo: 1.94 GRMS
- AFT +Y Interface Accel. Zo: 2.15 GRMS
- AFT -Y Interface Accel. Zo: 2 GRMS

Frequency (Hz)

Acceleration (G^2/Hz)
# Attenuated GRMS Summaries

## Fuselage/Carrier Interface GRMS Attenuations

<table>
<thead>
<tr>
<th>Fuselage/Carrier Interface</th>
<th>Input GRMS</th>
<th>Output GRMS Correlated Inputs</th>
<th>GRMS Attenuation</th>
<th>Output GRMS Uncorrelated Inputs</th>
<th>GRMS Attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFT STBD (Z)</td>
<td>6.95</td>
<td>2.18</td>
<td>68.63%</td>
<td>3.08</td>
<td>55.68%</td>
</tr>
<tr>
<td>FWD STBD (Z)</td>
<td>6.95</td>
<td>1.71</td>
<td>75.40%</td>
<td>2.22</td>
<td>68.06%</td>
</tr>
<tr>
<td>AFT PORT (Z)</td>
<td>6.95</td>
<td>2.56</td>
<td>63.17%</td>
<td>3.41</td>
<td>50.94%</td>
</tr>
<tr>
<td>FWD PORT (Z)</td>
<td>6.95</td>
<td>1.99</td>
<td>71.37%</td>
<td>2.36</td>
<td>66.04%</td>
</tr>
</tbody>
</table>

## Carrier/Cargo Interface GRMS Attenuations

<table>
<thead>
<tr>
<th>Carrier/Cargo Interface</th>
<th>Input GRMS</th>
<th>Output GRMS Correlated Inputs</th>
<th>GRMS Attenuation</th>
<th>Output GRMS Uncorrelated Inputs</th>
<th>GRMS Attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>FWD -Y (Z)</td>
<td>6.95</td>
<td>1.62</td>
<td>76.69%</td>
<td>1.99</td>
<td>71.37%</td>
</tr>
<tr>
<td>FWD +Y (Z)</td>
<td>6.95</td>
<td>1.34</td>
<td>80.72%</td>
<td>1.94</td>
<td>72.09%</td>
</tr>
<tr>
<td>AFT +Y (Z)</td>
<td>6.95</td>
<td>1.62</td>
<td>76.69%</td>
<td>2.15</td>
<td>69.06%</td>
</tr>
<tr>
<td>AFT -Y (Z)</td>
<td>6.95</td>
<td>0.91</td>
<td>86.91%</td>
<td>2.00</td>
<td>71.22%</td>
</tr>
</tbody>
</table>
Observations
Fuselage/Carrier/Cargo Example Problem

Mid to high frequencies: significant input environment attenuations were achieved at the Fuselage/Carrier interfaces with the STBD environment more attenuated due to Cargo manifested on the STBD side. As expected, additional input environment attenuations achieved at the Carrier/Cargo interfaces.

Low Frequencies: essentially no attenuations were observed for the fully correlated input case. Significant amplifications (10+dB) were observed at both the Fuselage/Carrier and Carrier/Cargo interfaces for the uncorrelated input case.

Impact of correlation states, specially to the lower frequency response, was found to be significant.
JPL Panel/Box-A Acoustic Test Data
JPL Acoustic Test
Panel + Box A

- Free-free aluminum panel (1.04 x 0.728 m; 17.1 kg) in acoustic chamber (w/ box-A configuration shown)
- Box-A (0.27 x 0.15 m footprint; 7.9 kg)
- Measured unloaded and loaded panel accelerations at locations shown
JPL Test Data

Notes

JPL test data were transmitted as time-histories. The data transmission included the panel unloaded accelerations and the panel/Box-A accelerations. Panel/Box-B accelerations were not transmitted given that the subject predictions were to be blind. Statistical signal processing was done at ASD at 4 Hz and 1 Hz resolutions. It was found that a 4 Hz resolution was inadequate to clearly define the panel fundamental bending modes from the test data. Therefore, a 1 Hz resolution was adopted for all comparisons.

Given the nature of the test, only the out-of-plane accelerations were considered to be of practical importance. This was due to the high in-plane stiffness of the panel coupled with the normal direction of the loading for the unloaded panel configuration (no box surfaces to induce lateral loads) resulted in negligible in-plane accelerations measurements.
Bare Panel Accelerations

A1Z through A8Z: 4 Hz Resolution

![Graph showing acceleration measurements for different tests with specified GRMS values.](image)
Bare Panel Accelerations
A1Z through A8Z: 1 Hz Resolution

- Test: A1Z, GRMS = 3.99
- Test: A2Z, GRMS = 4.23
- Test: A3Z, GRMS = 4.03
- Test: A4Z, GRMS = 4.43
- Test: A5Z, GRMS = 4.48
- Test: A6Z, GRMS = 4.24
- Test: A7Z, GRMS = 4.45
- Test: A8Z, GRMS = 4.19
Bare Panel Coherences
A56Z, A57, A58Z
Panel + Box-A Accelerations

A1Z through A8Z

Test+: A1Z; GRMS = 6.36
Test+: A2Z; GRMS = 7.51
Test+: A3Z; GRMS = 0.01
Test+: A4Z; GRMS = 7.38
Test+: A5Z; GRMS = 4.5
Test+: A6Z; GRMS = 3.4
Test+: A7Z; GRMS = 3.03
Test+: A8Z; GRMS = 2.9
Panel + Box-A Coherences
A56Z, A57Z, A58Z
Bare Panel vs Panel + Box-A

*Interface Accelerations (Average)*

Test: Panel (Average A5-A8); GRMS = 4.34

Test: Panel + Box-A (Average of A5-A8); GRMS = 3.51

![Graph showing interface accelerations for Bare Panel vs Panel + Box-A]
Bare Panel vs Panel + Box-A

Non-Interface Accelerations (Average)

Test: Panel (Average A1-A4); GRMS = 4.17

Test: Panel + Box-A (Average of A1-A4); GRMS = 7.1
Comparison of GMA Predictions to Panel + Box-A Test
Comparison of GMA to Test

Notes

It is important to note that the panel and Box-A finite element models (FEMs) utilized in this work were not test correlated. No attempts have been made to correlate the frequencies and mode-shapes of these FEMs to any test data.

The interface impedances were derived directly from these FEMs in as close as possible to the test accelerometer locations with a 1% uniform damping assumption. GMA predictions were made utilizing all 8 accelerometers.
FEM Frequencies/Test Accels

Overlay

Test: A1Z; GRMS = 3.99
Test: A2Z; GRMS = 4.23
Test: A3Z; GRMS = 4.03
Test: A4Z; GRMS = 4.43
Test: A5Z; GRMS = 4.48
Test: A6Z; GRMS = 4.24
Test: A7Z; GRMS = 4.45
Test: A8Z; GRMS = 4.19

FEM Frequencies

Frequency (Hz)

Acceleration (G^2/Hz)
GMA Predictions

A1Z through A8Z

Frequency (Hz)

Acceleration (G^2/Hz)

- GMA: A1Z; GRMS = 5.22
- GMA: A2Z; GRMS = 5.72
- GMA: A3Z; GRMS = 5.35
- GMA: A4Z; GRMS = 6.12
- GMA: A5Z; GRMS = 2.43
- GMA: A6Z; GRMS = 2.22
- GMA: A7Z; GRMS = 2.47
- GMA: A8Z; GRMS = 2.5
GMA Predictions
Coherences A56, A57, A58

![Graph showing coherences for frequencies from 10 Hz to 1000 Hz with different colors representing GMA:A56Z, GMA:A57Z, and GMA:A58Z.]
GMA vs Bare Panel Test

*Interface Accelerations (Average)*

- **Test:** Panel (Average A5-A8); GRMS = 4.34
- **GMA:** Panel + Box-A (Average of A5-A8); GRMS = 2.41
GMA vs Bare Panel Test

Non-interface Accelerations (Average)

Test: Panel (Average A1-A4); GRMS = 4.17
GMA: Panel + Box-A (Average of A1-A4); GRMS = 5.61
GMA vs Panel + Box-A Test

Interface Accelerations (Average)

Test: Panel + Box-A
(Average of A5-A8);
GRMS = 3.51

GMA: Panel + Box-A
(Average of A5-A8);
GRMS = 2.41
GMA vs Panel + Box-A Test

*Non-interface Accelerations (Average)*

![Graph showing non-interface accelerations for GMA vs Panel + Box-A Test.](image)

- **Test: Panel + Box-A** (Average of A1-A4); GRMS = 7.1
- **GMA: Panel + Box-A** (Average of A1-A4); GRMS = 5.61
GMA vs Panel + Box-A Test

Coherence A56Z

![Graph showing coherence between GMA and Panel + Box-A test frequencies. The x-axis represents frequency in Hz, ranging from 10 to 1000, and the y-axis represents coherence ranging from 10E-05 to 10E+00. The graph includes lines for Test+: A56Z and GMA:A56Z.](image-url)
JPL Panel/Box-B Acoustic Test Data
JPL Acoustic Test
Panel + Box B

- Free-free aluminum panel (1.04 x 0.728 m; 17.1 kg) in acoustic chamber (w/ box-B configuration shown)
- Box-B (0.47 x 0.25 m footprint; 20.3 kg)
- Measured unloaded and loaded panel accelerations and Box interface forces
Panel + Box-B Accelerations

A1Z through A8Z

Test+: A1Z; GRMS = 6.75
Test+: A2Z; GRMS = 4.95
Test+: A3Z; GRMS = 5.88
Test+: A4Z; GRMS = 4.41
Test+: A5Z; GRMS = 7.39
Test+: A6Z; GRMS = 7.32
Test+: A7Z; GRMS = 7.69
Test+: A8Z; GRMS = 7.09
Bare Panel vs Panel + Box-B

**Interface Accelerations (Average)**

- **Test: Panel (Average A1-A4); GRMS = 4.17**
- **Test: Panel + Box-B (Average of A1-A4); GRMS = 5.57**
Bare Panel vs Panel + Box-B

Non-interface Accelerations (Average)

Test: Panel (Average A5-A8); GRMS = 4.34
Test: Panel + Box-B (Average of A5-A8); GRMS = 7.38
Comparison of GMA Predictions to Panel + Box-B Test
Comparison of GMA to Test

Notes

It is important to note that the panel and Box-B finite element models (FEMs) utilized in this work were not test correlated. No attempts have been made to correlate the frequencies and mode-shapes of these FEMs to any test data.

The interface impedances were derived directly from these FEMs in as close as possible to the test accelerometer locations with a 1% uniform damping assumption. GMA predictions were made utilizing all 8 accelerometers.
GMA Predictions
A1Z through A8Z

- GMA: A1Z; GRMS = 2.83
- GMA: A2Z; GRMS = 2.4
- GMA: A3Z; GRMS = 2.86
- GMA: A4Z; GRMS = 2.54
- GMA: A5Z; GRMS = 7.5
- GMA: A6Z; GRMS = 10.86
- GMA: A7Z; GRMS = 11.74
- GMA: A8Z; GRMS = 7.74
GMA vs Bare Panel Test

Interface Accelerations (Average)

Test: Panel (Average A1-A4); GRMS = 4.17
GMA: Panel + Box-B (Average of A1-A4); GRMS = 2.67
GMA vs Bare Panel Test

Non-interface Accelerations (Average)

Test: Panel (Average A5-A8); GRMS = 4.34
GMA: Panel + Box-B (Average of A5-A8); GRMS = 9.64
GMA vs Panel + Box-B Test

**Interface Accelerations (Average)**

*Test: Panel + Box-B (Average of A1-A4); GRMS = 5.57*

*GMA: Panel + Box-B (Average of A1-A4); GRMS = 2.67*
GMA vs Panel + Box-B Test

Non-interface Accelerations (Average)

- **Test: Panel + Box-B**
  - Average of A5-A8
  - GRMS = 7.38

- **GMA: Panel + Box-B**
  - Average of A5-A8
  - GRMS = 9.64
Observations
GMA vs Test Comparisons

50-600 Hz: GMA predictions capture the major spectral characteristics of attenuations and amplifications of the measurements for both JPL test configurations.

20-50 Hz: GMA predictions show a lesser degree of attenuation of the input environments than test, specially for the panel/box-b configuration. This is to be further studied in conjunction with the force transducer measurements presented in the GMA force-limiting briefing.

600-2000 Hz: GMA predictions show that the high frequencies are well attenuated at the box interfaces (a physically expected result). The test shows essentially no high frequency attenuations. GMA predictions and test measurements are in agreement for the non box loaded interface locations. Further light will be shed on this with the force transducer measurements presented in the GMA force-limiting briefing.
Concluding Remarks

The GMA interface acceleration equation presents a multi-drive point, multi-axis equation allowing for input environment correlations for improved prediction of the interface accelerations.

Fundamental principle enabling the derivation is that the unloaded environment contains “finite energy” for driving the components added to the unloaded structure.

GMA does not require simplifying assumptions relative to the environment or coupling.

The mathematical tools utilized for the derivation are modal synthesis and random vibration theories.

GMA affords a rational and robust approach in analyzing complex structural systems.
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